M.Sc. Sem II

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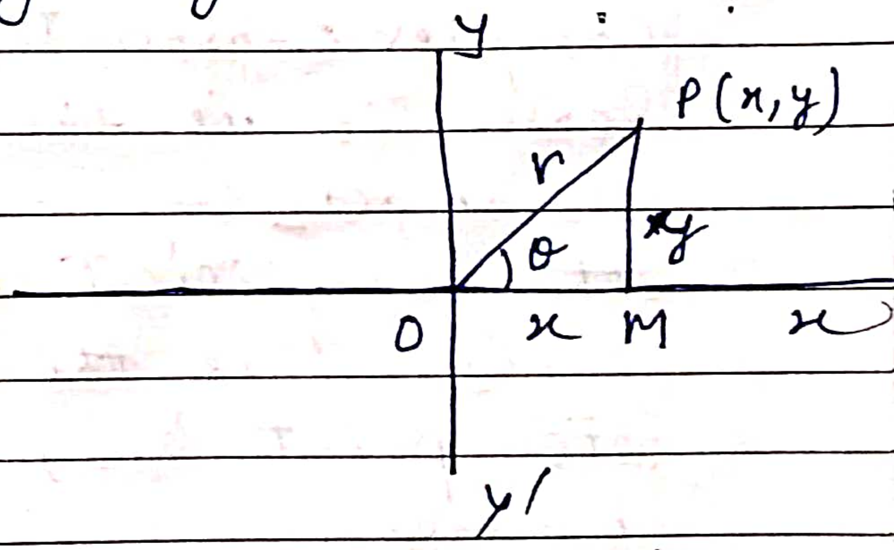
⇒ **Complex Plane** corresponding to each complex number $z = x+iy$, there is a unique point $P(x,y)$ in the xy plane which is known as complex plane or argand plane.
X-Axis is called real axis and y-axis is called imaginary axis
From figure

(1.) $|z| = |\overline{OP}|$

$= OP$

$= r =$

$= \sqrt{x^2+y^2}$



⇒ **Positionally Equal Numbers**
Two complex numbers z_1 & z_2 are said to be positionally equal if $|z_1| = |z_2|$
and $\arg|z_1| - \arg|z_2| = 2n\pi$
$n \in Z$.

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Power Series A series of the type

$$\sum_{n=0}^{\infty} a_n(z-z_0)^n = a_0 + a_1(z-z_0) + a_2(z-z_n)^2 + \cdots + a_n(z-z_0)^n + \cdots \quad \langle i \rangle$$

where $z_0, a_1, a_2 \cdots \cdots a_n$ are complex constants, called Power Series z_0 is the centre of series $a_0, a_1, a_2 \cdots a_n$ are known as the coefficient of the series.

Eq (1) is the Power Series about the point z_0, taking $z_0 = 0$ Eq (1) becomes

$$\sum_{n=0}^{\infty} a_n z^n = a_0 + a_1 z + a_2 z^2 + \cdots + a_n z^n + \cdots$$

Complex function $f(z)$ is a function of a complex variable z and is denoted by w
$\Rightarrow w = f(z) = u + iv$ where u and v are real functions of x and y and called real and imaginary parts respectively of $f(z)$.

Transcendental functions

The function involving trigonometric function, logarithmic function, exponential function and their algebra is known as transcendental function.

(1.) Exponential function

$$\exp(z) \text{ or } e^{z} = 1 + \frac{z}{1!} + \frac{z^2}{2!} + \cdots \frac{z^n}{n!} + \cdots \infty \quad \text{— (1)}$$

Properties of Exponential Equation

(a) $e^{z} = e^{x+iy} = e^{x}.e^{iy} = e^{x}(\cos y + i\sin y)$

$\Rightarrow |e^{z}| = |e^{x}| = e^{x} \quad (\because e^{x} > 0)$

and $\arg(e^{z}) = y$

b.) $e^{\bar{z}} = (e^{z})$

c.) e^{z} is periodic function having imaginary period $2\pi i$

d.) Series (1) is absolutely convergent in the whole complex plane

(e) Exponential form of $z = re^{i\theta}$.